# Performance Evaluation of Filter-bank based Spectrum Estimator

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***Abstract—***
**In this paper on attempt has been made to study the performance of Filter-bank based nonparametric spectral estimation. Several methods are available to estimate non parametric power spectrum. The band pass filter, which sweeps through the frequency interval of interest, is main element in power spectrum estimation setup. The filter-bank based spectrum estimation is developed and is applied to multi tone signal. The spectrum estimated based on filter-bank approach has been compared with conventional nonparametric spectrum estimation techniques such as Periodogram, Welch and Blackman-Tukey. It is observed that the filter-bank method gives better frequency resolution and low statistical variability. It is also found there is a tradeoff between resolution and statistical variability.**

***Index Terms—*** **Correlogram, Filter-bank, Frequency resolution, Periodogram, Spectral leakage etc.,**

## I. INTRODUCTION

In Signal processing, the nonparametric spectrum estimation plays an important role in determining perioditicity in random signals and thus a comprehensive elaboration of filter-bank based spectrum estimation techniques has been presented. In general, spectrum estimation can be categorized into direct and indirect methods. In direct method (usually recognized as frequency domain approach), the power spectrum is estimated directly from signal being estimated $x(n)$ . On the other hand, in indirect method, also known as time domain approach, the autocorrelation function of the signal being estimated $R_{xx}(k)$ is calculated. From this autocorrelation value, the power spectrum density can be found by applying the Discrete Fourier Transform on$R_{xx}(k)$. Another way to categorize spectrum estimation methods is by classifying them into parametric or non-parametric methods. Parametric method is basically model based approach [10]. In this method, a signal is modeled by Auto Regressive (AR), Moving Average (MA) or Auto Regressive Moving Average (ARMA) process. Once the signal is modeled, all parameters of the underlying model can be estimated from the observed signal. Estimator based on parametric method provides higher degree of detail.

The disadvantage of parametric method is that if the signal is not sufficiently and accurately described by the model, the result is less meaningful. Non Parametric methods, on the other hand, do not have any assumption about the shape of the power spectrum and try to find acceptable estimate of the power spectrum without prior knowledge about the underlying stochastic approach. The following sub-sections give review on some of the spectrum estimation methods.

### *A. Periodogram*

The most commonly known spectrum estimation technique is periodogram, which is classified as a non parametric estimator. The procedure starts by calculating the Discrete Fourier Transform (DFT) of the random signal being estimated, followed by taking the square of it and then dividing the result with the number of samples *N*.

There are five common nonparametric PSE available in the literature: the periodogram, the modified periodogram, Bartlett’s method, Blackman–Tukey, and Welch’s method. However, all these nonparametric PSE, s are modifications of the classical periodogram method introduced by Schuster.

The periodogram is defined as [2]

$$\hat{S}_{xx}(k)=\frac{1}{N}\left|\sum_{n=0}^{N-1}x(n)e^{-\frac{2\Pi jkn}{N}}\right|^2 \qquad (1)$$

It is known that the periodogram is asymptotically unbiased but inconsistent because the variance does not tend to zero for large record lengths. One can show [10], that the Variance on the periodogram $\hat{S}_{xx}(k)$ of an ergodic weakly stationary signal $x(n)$ for $n=0:N-1$is asymptotically proportional to $S^2{}_{xx}(k)$, the square of the true power at frequency bin. The periodogram uses a rectangular time-window, a weighting function to restrict the infinite time signal to a finite time horizon, the modified periodogram uses a nonrectangular time window [3].

A way to enforce a decrease of the variance is averaging. Bartlett’s method divides the signal of length

$N$ into $K$ segments of length $L = \frac{N}{K}$ each. The periodogram method is then applied to each of the $K$ segments. The average of the resulting estimated power spectra is taken as the estimated power spectrum. One can show that the variance is reduced by a factor $K$, but the spectral resolution is also decreased by a factor $K$, [2]. The Welch method eliminates the tradeoff between spectral resolution and variance in the Bartlett method by allowing the segments to overlap [4].

*B. Blackman-Tukey method (Windowed Correlogram)*

Blackman-Tukey method is a variant of correlogram that computes the approximated autocorrelation $R_{xx}(k)$ and later applies a suitable window function $w[k]$. The power spectra density is then obtained by computing the Fourier Transform of windowed auto-correlation sequence [10].

Blackman-Tukey method is generally described as follows

$$S_{xx}^{BT}(e^{jw}) = \sum_{k=-L}^{L} \hat{R}_{xx}(k)\, w(k) \exp(-jwk) \tag{2}$$

And its frequency domain representation is given by

$$S_{xx}^{BT}(e^{jw}) = \frac{1}{2\pi} W(e^{jw}) S_{xx}^{c}(e^{jw}) \tag{3}$$

from equation (3) the Blackman-Tukey method can actually be viewed as a process of smoothing the correlogram by convolving the correlogram with the kernel of selected window. This smoothing process plays an important role to reduce the bias of estimated PSD but this convolution process would reduce the frequency resolution. The amount of frequency resolution reduction is strongly related to the size of the main lobe of the window kernel. Section II talks about filter bank approach to spectrum estimation technique. Section III focuses the performance analysis of the proposed techniques in comparison with the conventional techniques.

## II. SPECTRUM ESTIMATION AS A FILTER BANK ANALYSIS

From the perspective of spectrum estimation, a filter bank can be considered as an array of band pass filters that separates the input signal into several frequency components, each one carrying a single frequency sub-band [6]. The filter banks are usually implemented based on single prototype filter, which is a low pass filter. This low pass filter is normally used to realize the zero-th band of the filter bank while filters in the other bands are formed through the modulation of the prototype filter [9]. Figure 2.6 illustrates the main idea of filter bank concept. This section basically tries to explore the filter bank paradigm in spectrum estimation.

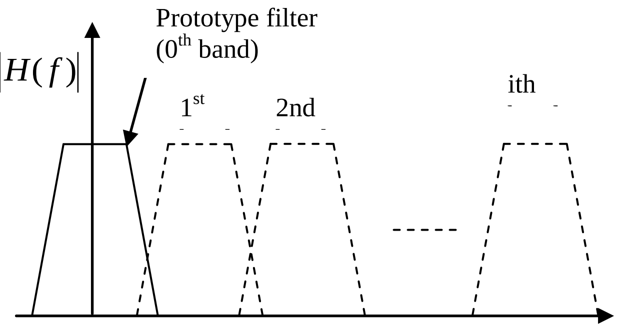


Figure.1 The filter bank concept.

*A. Periodogram spectral estimator realization through filter banks*

Spectrum estimation is about finding the power spectrum density (PSD) of a finite sample set $\{x(n), n = 1,2,\dots\dots N\}$ for frequency $|\omega| \le \pi$. The classical approach to spectrum estimation is to use Fourier transforms to obtain a Periodogram, given as [17]:

$$S_{xx}^{p}(e^{j2\pi f}) = \frac{1}{N}\left|\sum_{n=1}^{N} x(n) e^{-j2\pi f n}\right|^2 \tag{4}$$

for any given frequency $f_i$, (4) is written as:

$$\begin{aligned} S_{xx}^{p}(e^{j2\pi f_i}) &= \frac{1}{N}\left|\sum_{n=1}^{N} x(n) e^{-j2\pi f_i n}\right|^2 \\ &= \frac{1}{N}\left|\sum_{n=1}^{N} x(n) e^{j2\pi f_i (N-n)}\right|^2 \end{aligned} \tag{5}$$

it should be noted that (5) is possible since $\left|e^{(j2\pi f_i N)}\right| = 1$. By introducing new variable $k = N - n$, equation (5) is written as:

$$\begin{aligned} S_{xx}^{p}(e^{j2\pi f_i}) &= \frac{1}{N}\left|\sum_{k=0}^{N-1} x(N-k) e^{j2\pi f_i n}\right|^2 \\ &= \frac{1}{N}\left|\sum_{k=0}^{N-1} h_i(k) x(N-k)\right|^2 \end{aligned} \tag{6}$$

where $h_i(k) = \frac{1}{\sqrt{N}} e^{(j2\pi f_i k)}$ for $k = 0,1,2,\dots\dots N-1$

by observing the summation within the magnitude operation in (6) and the summation expressed as:

$$y(N) = \sum_{k=0}^{N-1} h_i(k) x(N-k) \tag{7}$$

(7) is actually the truncated convolution sum at particular point N, which is again written as general convolution sum at the same point associated with a linear causal system by padding

$h_i(k)$ with zeros[43]. Then (7) is rewritten as

$$y_a(N) = \sum_{k=0}^{\infty} h_i(k)x(N-k) \quad (8)$$

with

$$h_i(k) = \begin{cases} w(k)e^{(j2\pi f_i k)} & for\, k = 0,1,2,\ldots\ldots N-1 \\ 0 & otherwise \end{cases} \quad (9)$$

and window function $w(k) = 1/\sqrt{N}$. It is clear that (8) represents as passing N samples through a filter having impulse response $h_i(k)$ and then taking only single sample of the filtered signal at point N. Based on this perspective, the frequency response of the linear filter having impulse response $h_i(k)$ is

$$H_i(\omega) = \sum_{k=0}^{\infty} h_i(k)e^{-j\omega k} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{j(\omega_i - \omega)} = \frac{1}{\sqrt{N}} \frac{e^{j(\omega_i-\omega)N} - 1}{e^{j(\omega_i-\omega)} - 1} \quad (10)$$

finally gives

$$H_i(\omega) = \frac{\sin[N(\omega_i - \omega)/2]}{\sqrt{N}\sin[\omega_i - \omega)/2]} \exp[\, j(\frac{N-1}{2})(\omega_i - \omega)] \quad (11)$$

If $w(k)$ in (9) is taken to be a prototype FIR (Finite Impulse Response) low pass filter, then $h_i(k)$'s will constitute a bank of band pass filters centered at frequencies $f_i$ s. This filter bank is constructed by modulating the prototype filter. By considering (4)-(11), the periodogram estimate at particular frequency point $f_i$ is obtained by passing the received samples through the band pass filter centered at $f_i$. The power calculation of this estimate is performed based only on a single sample of the output of the filter from (8). The entire periodogram estimates can then be related to the output of several filters in the filter bank constructed by modulating a single prototype filter $w(k)$. For the case of simple periodogram, the window function $w(k)$ is rectangular with $w(k) = 1/\sqrt{N}$ . As it is clear from (11), the frequency response of filter based on prototype filter having rectangular

window as its impulse response would have significant level of side lobes. This is actually the main reason why the periodogram estimates have high side lobe or large leakages. This problem can be alleviated by replacing the rectangular window with a window function with a taper that smoothly decays on both sides to obtain a prototype filter with much smaller side lobes. A few popular windows are Hanning, Kaiser and Blackman [8]

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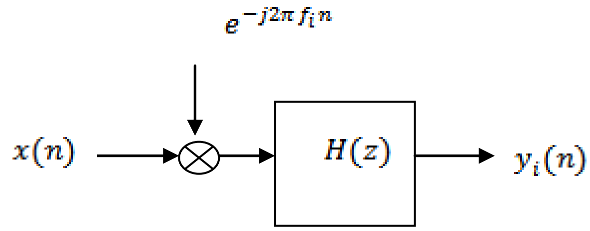



Figure.2 The demodulation of received signal

From the above, the implementation of a spectrum estimator using filter bank for signal analysis is clear, namely by passing an input signal through a bank of filters. The output power of each filter is a measure of the estimated power over the corresponding sub-band. Hence the power spectral density (PSD) estimate of *i*-th sub band of the filter bank is represented as [16]:

$$\hat{S}\left(\frac{i}{N}\right) = avg[|y_i(n)|^2] \quad (12)$$

In (12), avg [] describes time average operator while $y_i(n)$ is the output signal of $i^{th}$ sub band filter.

## III. Results and Analysis

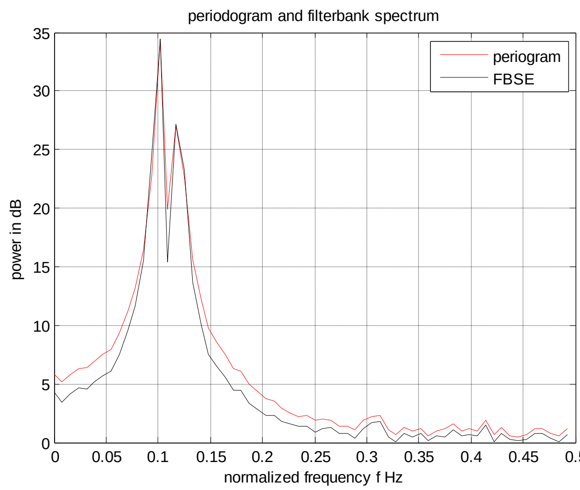
Consider a signal having two closed frequencies embedded in white noise i.e. $x(n) = \sin(2\pi f_1 t) + \sin(2\pi f_2 t) + e(n)$, where $f_1 = 0.1, f_2 = 0.12$ and $e(n)$ is white noise having zero mean and unit variance. The performance of the estimation techniques is mainly evaluated with respect to three different parameters:

- Frequency resolution
- Variance of the estimated power spectrum density (PSD)

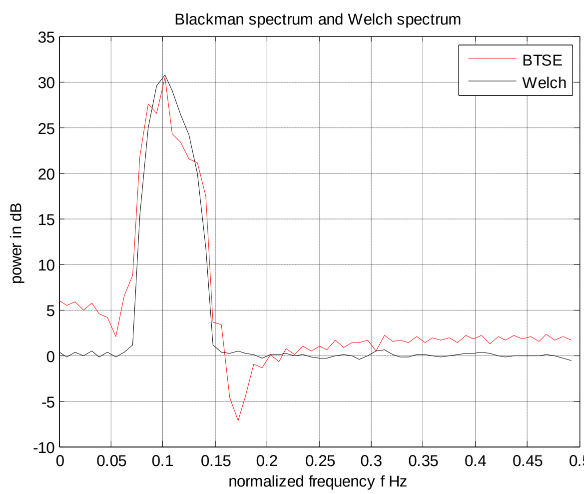
Fig. 6 depicts Periodogram, Blackman-Tukey, and Welch approach as well as filter-bank based estimates for the case of a signal having two closed frequencies. In these figures, the number of samples used in the experiment is N=128,256 and 512. For the purpose of this experiment, the Welch approach divides the received samples into M=4 segments of K=N/M samples. Two consecutive segments overlap to one another by 50%. Before performing the averaging process, Hamming window is applied on each segment. As in the case of Blackman-Tukey approach, a triangular window is used having its length K=N/2. The number of bandpass filter in filter-bank implementation is denoted by K.

In Fig.6, assumed K=1, where both the filter-bank based spectrum estimate and periodogram based spectrum estimate have good frequency resolution with high statistical variability. In Fig. 7 and Fig.8, assumed K=4, where the filter-bank spectrum estimate is much better than periodogram having low variability while maintaining acceptable frequency resolution. It is also observed that the level of estimated power in unoccupied band for Welch approach is higher than for simple periodogram meaning that the Welch approach offers poorer rejection in the unoccupied band. This is understandable since

Welch approach divides the received samples into several segments with lower number of samples before estimating each segment. Hence Welch approach offers very low variability and poor frequency resolution. The Blackman-Tukey method is also good estimates to power spectrum having low variability and moderate frequency resolution. Finally, we noticed that when K=4 and N=256 and 512, with respect to variance and frequency resolution, the filter bank approach most preferable to estimate power spectrum among the above techniques.

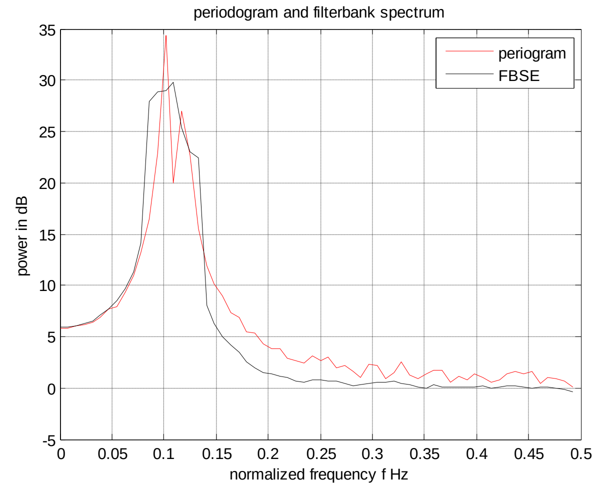



(a)

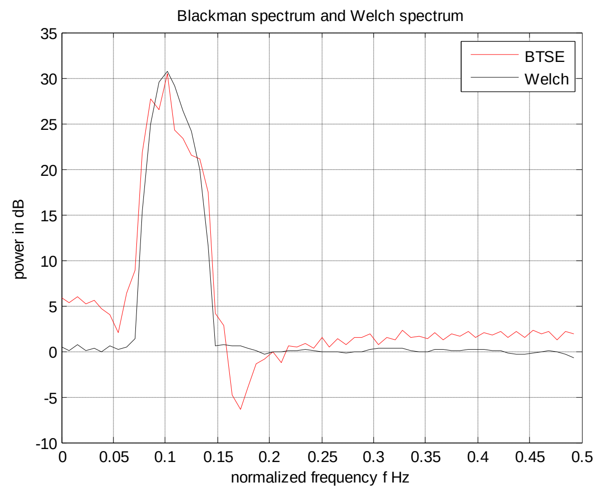



(b)

Figure. 6 (a) Periodogram and filterbank based spectrum (b) blackman tukey an Welch based spectrum (K=1, N=128)

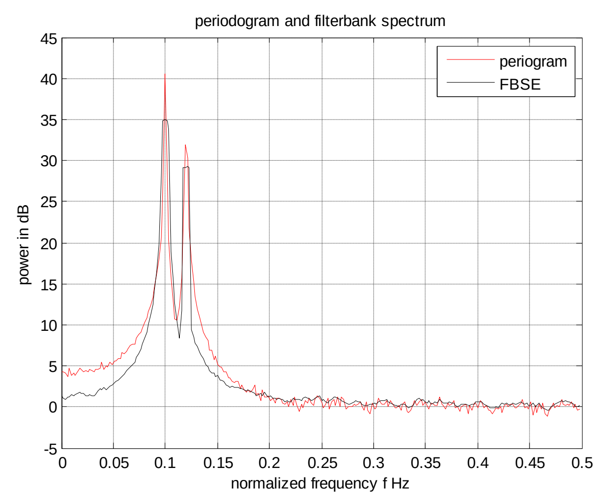



(a)

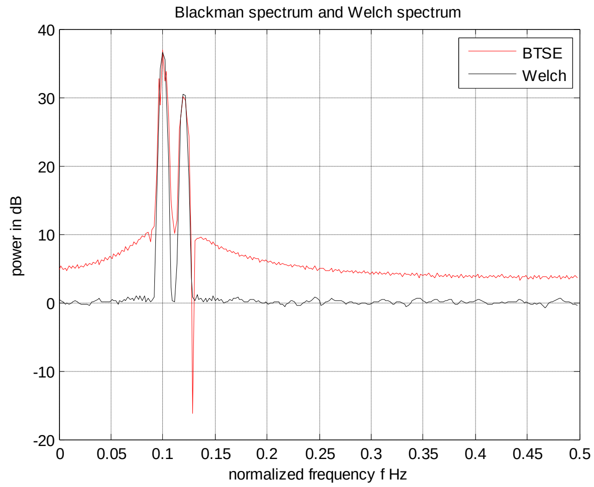



(b)

Figure.7. (a) Periodogram and filterbank based spectrum (b) blackman tukey an Welch based spectrum (K=4, N=128)




(a)




(b)

Figure.8. (a) Periodogram and filterbank based spectrum (b) blackman tukey an Welch based spectrum (K=4, N=512)

**Table I**: the number of bandpass filters K=1, In Welch approach (hamming window, 50% overlap and N/4 segments) and in Black-man Tukey (hamming window of length N/2)

| Variance | N=128 | N=256 | N=512 |
|---|---|---|---|
| Periodogram | 1.2156e+005 | 1.3194 e+005 | 5.2874e+005 |
| Filter-bank approach | 1.2409e+005 | 1.5029e+005 | 5.2829e+005 |
| Black-man tukey | 0.3110e+005 | 0.7978e+005 | 1.5834e+005 |
| Welch | 0.4704e+005 | 0.8525e+005 | 1.7550e+005 |

**Table II:** the details are same as in Table I, except change in the number of bandpass filters K=4

| Variance | N=128 | N=256 | N=512 |
|---|---|---|---|
| Periodogram | 1.2056e+005 | 1.3177e+005 | 5.3098e+005 |
| Filter-bank approach | 0.3772e+004 | 0.7844e+005 | 1.4257e+005 |
| Black-man tukey | 0.3047e+004 | 0.7946e+005 | 1.5844e+005 |
| Welch | 0.4631e+004 | 0.8513e+004 | 1.7584e+005 |

The variance analysis among these techniques as follows

- From Table I and Table II, for length of samples N=128,256 and 512, the Blackman-Tukey approach offers low variability than other methods.
- From Table II, for a record length of N=256 and 512, the FBA offers low variability without scarifying frequency resolution than other methods.

The frequency resolution among these techniques as follows

- From Figures, for record length of N=256, the Welch approach does not resolve two closed frequencies. Its resolving capability increases with data record length.
- From Figures, when K=4, N=256 and N=512, the Filter-bank approach resolve two closed frequencies.

## IV. Conclusion

In this paper on attempt has been made to develop and implement filter bank based nonparametric spectral estimation technique. The proposed technique has been subjected to multi tone signal, and estimated the spectral components. The performance of proposed technique has been compared with the conventional methods of nonparametric spectrum estimation such as periodogram, Welch and Blackman-Tueky. It is observed that the FBA method produce spectral estimates with high resolution and low statistical variability at expense of increased the number of bandpass filters. Hence, there is tradeoff between resolution and statistical variability. The studies show that the Filter bank based spectrum estimation is simple, offers great flexibility, reconfigurability and adaptability.

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